the assumptions really work in all the turbulent flows. Hence, generalization in turbulence is very risky. In other words, the quantity required has to be measured.

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Conical, Separated Flows with Shock and Shed Vorticity

F. Marconi*
Grumman Corporate Research Center
Bethpage, New York

Introduction

THIS investigation deals with the highly vortical flow about circular cones at supersonic speeds and high angle of attack. The author has shown that the shock system of these flows can cause separation and a spiral vortex near the lee plane of the cone.[1] Earlier, Salas[2] showed two-dimensional Euler solutions where shock vorticity had also induced separation. The numerical investigation of Ref. 1 showed that the crossflow shock can produce large entropy gradients and vorticity sufficient to cause separation and a spiral vortex. In addition to shock vorticity, a separating boundary layer can shed vorticity into an otherwise irrotational flow. This phenomenon has been studied with inviscid models for many years. Smith[3] developed a model for shedding vorticity from the surface of a smooth body into the flowfield. The present author[4] was the first to use this model in conjunction with the Euler equations to shed vorticity from both primary and secondary crossflow separation points. An extension of the vorticity shedding model of Ref. 3 to the situation of supersonic crossflow is used in the present investigation. In the case of shock-induced separation, the solution to the Euler equation is unique in that the separation point location is computed along with the shock system and flowfield. On the other hand, the separation point must be prescribed in the case of shed vorticity. There exists a range of separation point locations corresponding to varying amounts of vorticity being shed. This Note presents the results of an investigation of the relationship between shock-vorticity-produced separation and shed-vorticity-produced separation.

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*Senior Staff Scientist. Associate Fellow AIAA.

Discussion

This study was conducted by numerically evaluating Euler solutions for the flow about a 5 deg cone varying the specified separation point location. The freestream conditions were fixed at $M_\infty = 4.25$ and $\alpha = 12.35$ deg. The flow is conical so that only the crossflow plane (i.e., any plane normal to the cone axis) need be considered. This plane intersects the separation line at a point which will be referred to as the separation point. At the large angle of attack considered here the flow in the crossflow plane is supercritical and so a crossflow shock is present. With no vorticity shed from the cone surface, this crossflow shock is strong enough to produce enough vorticity to cause separation. In addition, shedding vorticity from the cone surface does not eliminate the crossflow shock so that both sources of vorticity are present. Figures 1 and 2 show the crossflow streamlines and isobars, respectively, for the flow with no vorticity shed from the body. Separation for this case is computed to be at $\theta_s = 151.3$ deg from the wind plane. Figure 1 indicates that the separating streamlines leave the surface at a large angle (57 deg in this particular case) relative to the surface. When only shock vorticity is present there is no jump in crossflow velocity at the separation point, consistent with the fact that no vorticity is being shed from the surface. In the case of the shock vorticity alone, the crossflow stagnates on both sides of the separating streamline, and this streamline leaves the surface at a large angle relative to it. Smith[3] proved that in order to shed vorticity into an otherwise irrotational flow there must be a jump in crossflow velocity at the separation point; the crossflow stagnates only on the lee side of the separating steamline. It should be pointed out that in the computational results presented here, all crossflow shocks are captured. Figure 2 indicates that the shock is captured very sharply (see the closely spaced isobars). Additionally, these captured shock results compare well with the shock fit results of Ref. 1.

The model used to shed vorticity from the cone surface follows the work of Smith.[3] The model is implemented in the computational scheme by using a double grid point at the separation point. One grid point is assumed on the flow side of the vortex sheet and the other is assumed on the body side. In the case of subsonic crossflow the conditions on the flow side of the sheet are computed as any other body point, satisfying only the flow tangency condition. The crossflow is assumed to stagnate on the body side of the sheet; with the pressure given from the flow side of the sheet all conditions at this point can be computed. These conditions result in the sheet leaving the body tangentially. If the crossflow approaching the separation point is supersonic the model used must be changed. Smith[3] argued that if the separating streamline leaves a smooth surface at a finite angle, then the flow must stagnate on both sides of the separation point and no vorticity will be shed into the flowfield. This argument is true if the flow approaching the separation point is subsonic. However, when the separating flow is supersonic and the separating sheet leaves the body at an angle relative to it, then there can be a jump in crossflow velocity (i.e., vorticity shed). In a locally supersonic flow a slope discontinuity results in a shock, not a stagnation point as in the subsonic case. The new model involves an oblique shock at the separation point unless the deflection is too large, in which case a normal shock is formed and the flow is separated in a locally subsonic flow. The modifications to the computation are as follows: the flow side of the sheet is also on the low-pressure side of the oblique shock at separation so that the pressure there cannot be used on the body side of the sheet. The crossflow on the body side of the sheet is still stagnated with the pressure there being extrapolated from the lee side of separation. With the conditions at the double point defined, no differences are taken across the separation point on the body. Once the sheet leaves the body no provisions are made for it and it is captured over a few mesh points. Reference 4 gives more detail on this procedure.

Figures 3 and 4 show the results for the most windward separation point location computed ($\theta_s = 115$ deg). Figure 3 shows the crossflow streamlines. The secondary separation shown is due to a strong reverse crossflow shock (see Fig. 4); note the third vortex off the surface due to the interaction of the two separations. In Fig. 4, the isobars are shown and they indicate an oblique shock at the specified primary separation pont. The shock is due to the separating sheet leaving the cone surface at an angle relative to it. A similar shock/sheet configuration has been noted experimentally.[5] In the situation of Figs. 3 and 4, the supersonic crossflow passes through the oblique shock and moves outward along the separating streamline. The flow on the lee side of the sheet stagnates at the separation point. With separation specified at $\theta = 115$ deg, the jump in crossflow velocity at the separation point indicates that significant vorticity is being shed from the surface. A comparison of Figs. 1 and 3 shows that the extent of the vortical regions are comparable, whereas the two sources of vorticity are very different.

The relationship between shock vorticity and shed vorticity for conical separated flow is made clear by considering Fig. 5. The figure shows the jump in crossflow velocity (Δq_{CF}) vs separation point location. The jump in crossflow velocity is directly related to the vorticity shed into the flowfield from the separation point. Figure 5 represents all possible Euler solutions for the flow about this cone at these freestream conditions and one specified separation point. No solutions could be found for $\theta_s > 151.3$ deg or $\theta_s < 115$ deg. The experimental

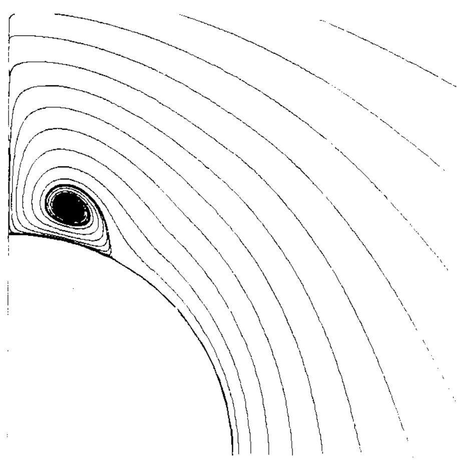

Fig. 1 Crossflow streamlines on the 5 deg circular cone ($M_\infty = 4.25$, $\alpha = 12.35$ deg) separation due to shock vorticity alone at $\theta = 151.3$ deg.

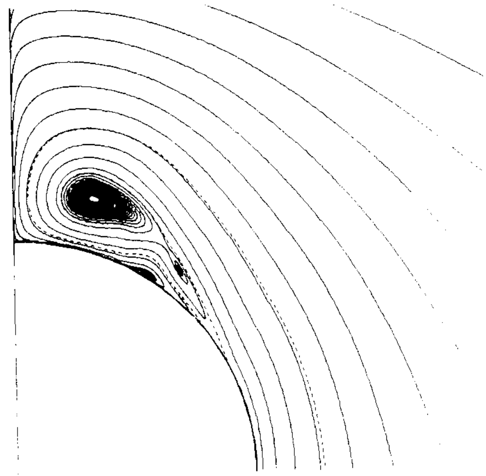

Fig. 3 Crossflow streamlines on the 5 deg circular cone ($M_\infty = 4.25$, $\alpha = 12.35$ deg) separation forced at $\theta = 115$ deg.

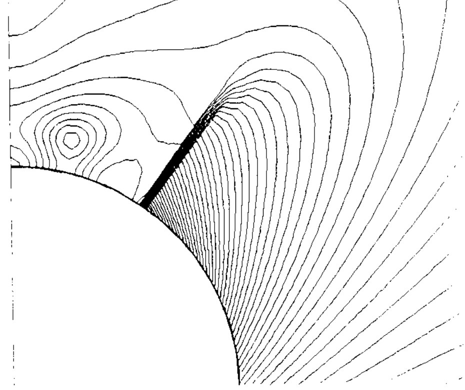

Fig. 2 Isobars on the 5 deg circular cone ($M_\infty = 4.25$, $\alpha = 12.35$ deg) separation due to shock vorticity alone at $\theta = 151.3$ deg.

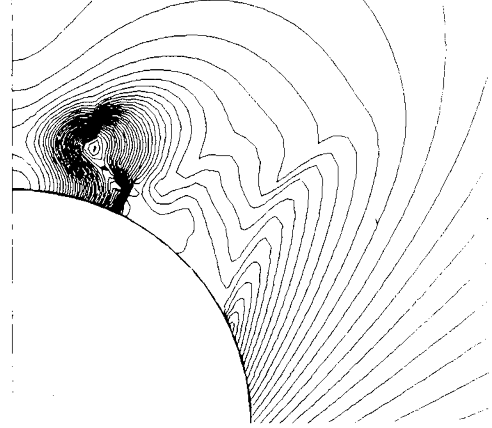

Fig. 4 Isobars on the 5 deg circular cone ($M_\infty = 4.25$, $\alpha = 12.35$ deg) separation forced at $\theta = 115$ deg.

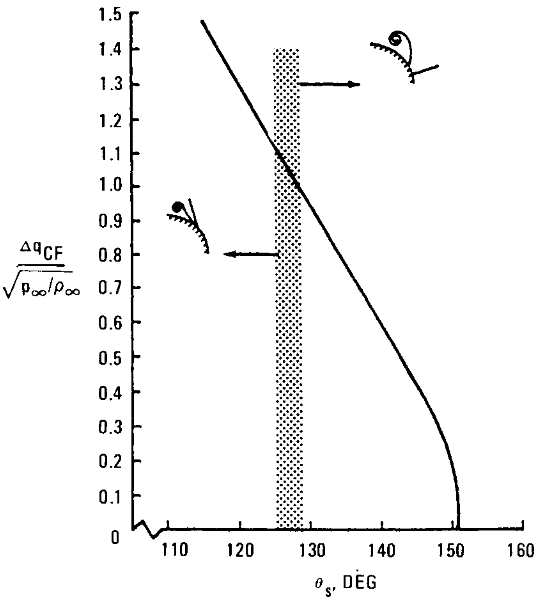

Fig. 5 Vorticity shed into the flowfield as a function of separation point location.

data of Ref. 5 indicate primary separation at 120 deg and secondary separation at 160 deg for this case. The shock configuration transition from an oblique crossflow shock to a detached normal crossflow shock occurs at about $\theta_s \approx 128$ deg (indicated by the shaded area in Fig. 5). It should be pointed out that the jump in velocity at the separation point in the oblique shock cases was computed by subtracting the oblique shock velocity jump from the numerical results. Thus, the jumps in velocity in Fig. 5 represent the jumps across the vortex sheet at separation. The figure shows that this velocity jump goes to zero smoothly as the separation point location due to shock vorticity alone is approached ($\theta_s = 151.3$ deg).

Conclusion

The results of this analysis indicate that there is a relationship between separation produced by shock vorticity and shed vorticity and that both sources of vorticity may be important. The relationship is inferred because the jump in velocity (i.e., vorticity shed) smoothly goes to zero as the shock vorticity separation point location is approached. In fact, it would seem that separation due to shock vorticity alone can be considered a limiting solution of the set of solutions in which vorticity is shed from the surface. In this limiting solution the vorticity shed is zero.

Acknowledgment

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Supersonic Laminar Flow Development in a Square Duct

D. O. Davis* and F. B. Gessner†
University of Washington, Seattle, Washington
and
G. D. Kerlick‡
Informatics General Corporation
NASA Ames Research Center
Moffett Field, California

Introduction

THE nature of laminar flow along a streamwise corner has been previously investigated, primarily for the limiting case of zero-pressure gradient flow along two semi-infinite plates that intersect at 90 deg. The problem is typically analyzed by means of matched asymptotic expansions applied to reduced forms of the conservation equations in order to determine far-field conditions, which then serve as boundary conditions when the conservation equations are applied to the corner layer. The results for both incompressible[1-3] and compressible[4,5] flow show that a crossflow exists in the far field when it is directed toward the corner in the viscous layer immediately adjacent to each plate. This crossflow increases in magnitude as the Mach number increases from subsonic to supersonic values, and is larger in magnitude when the walls are adiabatic, rather than highly cooled.[4,5]

Associated with this behavior is an outwardly directed crossflow along the corner bisector that is not part of a closed vortical pattern.[1-6] Whether or not this crossflow leads to an outward bulging of isovel (axial velocity) contours in the cross plane or, alternatively, total pressure contours, has been the subject of recent controversy. The results of the earlier analyses by Refs. 1-4 all show that isovel contours are undistorted by an outwardly directed crossflow along the corner bisector. A more recent analysis by Nomura[7] has shown, however, that these contours will always bulge outward if the plate intersection angle is between 0 and 180 deg.

This type of distortion was first observed experimentally by Zamir and Young[8] for nominally zero pressure gradient, incompressible flow along a 90-deg corner formed by two intersecting plates with an airfoil-shaped leading edge. In a subsequent study, El-Gamal and Barclay[9] utilized the same corner configuration, but with a sharp (6-deg wedge) leading edge, and did not observe distorted isovel contours in the corner region. On the basis of these results, the authors attributed the appearance of contour distortion in Zamir and Young's experiments to the airfoil-shaped leading edge used in that study.

In a subsequent paper, Zamir and Young[10] agree with this point of view, but argue that the distortion-free results observed by El-Gamal and Barclay[9] were influenced by favorable pressure gradient effects that had a stabilizing in-

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*Graduate Research Assistant, Department of Mechanical Engineering.

†Professor, Department of Mechanical Engineering. Member AIAA.

‡Research Scientist, Applied Computational Aerodynamics Branch. Member AIAA.